JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 16, No. 3, May-June 1993

Enhancement of Endurance Performance by Periodic Optimal Camber Control

Gottfried Sachs* and Rainer Mehlhorn†
Technische Universität München, 8000 Munich 2, Germany

A new method for endurance maximization is proposed. This method consists of the periodic optimal control of variable camber in a coordinated process with a corresponding control of throttle and elevator. It is shown that the proposed method can significantly increase the endurance when compared with the best steady-state flight. Using camber as a control provides an efficient means for improving the lift/drag ratio at each flight condition during the unsteady phases of periodic optimal endurance cruise. Engine considerations include fuel consumption during idling phases. They also address a reduction of the number of engine cycles in regard to alternate operating at a high and low thrust setting. The optimal control technique used is based on the minimum principle and the method of multiple shooting. Necessary conditions for optimality, including optimal control of variable camber setting, are analyzed.

Nomenclature

C_D = drag coefficient
C_L = lift coefficient
D = drag
g = acceleration due to gravity
H = Hamiltonian
h = altitude
J = performance criterion
L = lift
M = Mach number
m = mass
S = reference area
$\bar{S}$ = switching function
T = thrust
t = time
V = speed
α = angle of attack
γ = flight-path angle
δ_T = throttle setting
δ_{VCi} = variable camber setting, $i = 1, 2$
λ = Lagrange multiplier
ρ = atmospheric density
σ = fuel consumption factor
τ = normalized time, $\tau = t/t_{cyc}$

Introduction

MAXIMIZING endurance is a basic optimal control problem for aircraft similar to maximizing range, with flight time per fuel consumed as the performance criterion instead of distance traveled.

According to classical aircraft cruise, maximum endurance flight is traditionally considered as a steady-state cruise at constant speed or altitude with a rectilinear trajectory. However, recent papers[1-22] show that the endurance and range performance of aircraft can be enhanced by a periodic type of cruise consisting of alternating climbing and sinking flight phases basically produced by correspondingly high and low thrust settings.

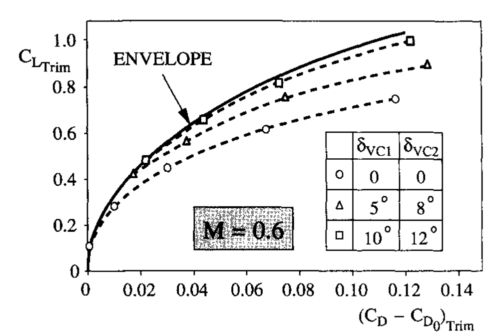
In papers presented so far on optimal periodic cruise, two controls are considered. These controls are throttle setting and lift coefficient (or elevator), respectively. A paper useful for control harmonization issues studies optimization with wing-sweep control.[23]

It is the purpose of this paper to propose a new method for maximizing endurance. This method introduces variable camber as a further control. Variable camber enables an advantageous effect on drag, such that it is possible to improve the lift/drag ratio for each flight condition during the unsteady phases of an optimal period by appropriately adjusting the profile of a wing.

Variable Camber Considerations

Changing the profile of a wing for the best adaptation to the performance requirements of different flight maneuvers is an aerodynamic concept known as "variable camber." This concept, which has recently received great interest, provides an effective means for improving the lift-drag characteristics of an aircraft.[24-27] Varying camber during flight enables the wing profile to be changed, such that it is adapted in the best manner to every angle of attack and Mach number combination. An example is presented in Fig. 1 that shows drag characteristics of an aircraft for various camber settings. This figure makes evident three points of particular significance for the utilization of variable camber as proposed in this paper for periodic optimal endurance cruise:

1) Basically, drag can be effectively controlled by varying camber such that the achievable lift/drag ratios may be signif-



	δ_{VC1}	δ_{VC2}
○	0	0
△	5°	8°
□	10°	12°

Fig. 1 Effect of variable camber on drag characteristics (high-performance aircraft, $M = 0.6$), from Ref. 23.

Presented in part as Paper 91-2670 at the AIAA Guidance, Navigation, and Control Conference, New Orleans, LA, Aug. 12–14, 1991; received Oct. 21, 1991; revision received May 20, 1992; accepted for publication Aug. 17, 1992.

*Director, Institute of Flight Mechanics and Flight Control. Associate Fellow AIAA.
†Research Assistant.

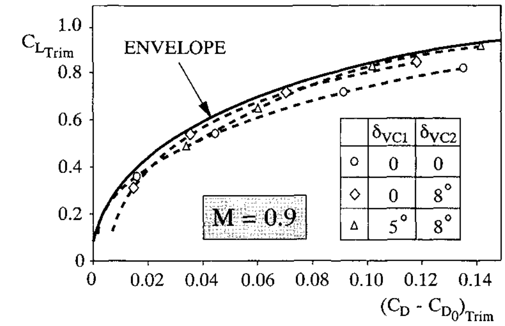


Fig. 2 Effect of variable camber on drag characteristics (high-performance aircraft, $M = 0.9$), from Ref. 23.

icantly improved for the whole range of angles of attack when compared with a fixed camber wing.

2) The improvements are particularly significant for larger angles of attack. This may be of particular advantage for periodic optimal endurance flight that shows phases where angles of attack and lift coefficients are rather large.[19]

3) A further point concerns the Mach number regions where variable camber may be most effective and its effectiveness reduced. The results presented in Fig. 1 concern a comparatively low Mach number. For a higher subsonic Mach number, the influence of variable camber on drag characteristics is shown in Fig. 2. Here, the effectiveness of variable camber is significantly reduced due to the influence of compressibility on drag. Since the lower Mach number range is of particular interest for periodic optimal endurance cruise,[19] the effectiveness of variable camber existing here may be fully utilized.

There are different ways wing camber variation may be realized.[24–27] A basic technique would be a combination of flaps at the trailing and leading edges of a wing (see Fig. 3). Another more sophisticated method consists of a segmentation of the wing, as also illustrated in Fig. 3. As may be seen, such a wing displays basically conical camber and a wide variation of shapes may be possible. The ultimate technique would be a procedure that enables a smooth change of the whole profile.

Problem Formulation

The optimal control problem may be formulated as maximizing the flight time per fuel consumed. A corresponding performance criterion is

$$J = \frac{t_{cyc}}{(m_f)_{cyc}} \tag{1}$$

where t_{cyc} is the time of one period of the trajectory and $(m_f)_{cyc}$ the fuel consumed. A period may be considered a basic element of the whole trajectory.

The performance criterion is subject to the equations of motion. By using the normalized time $\tau = t/t_{cyc}$ as an independent variable, the equations of motion may be written as

$$\frac{dV}{d\tau} = t_{cyc}\left(\frac{T}{m} - \frac{D}{m} - g \sin\gamma\right)$$

$$\frac{d\gamma}{d\tau} = t_{cyc}\left(\frac{L}{mV} - \frac{g}{V}\cos\gamma\right)$$

$$\frac{dh}{d\tau} = t_{cyc} V \sin\gamma$$

$$\frac{dm_f}{d\tau} = t_{cyc} f(V,h;\delta_T) \tag{2}$$

The aerodynamic model for the lift and drag force is

$$L = C_L(\rho/2)V^2S$$

$$D = C_D(\rho/2)V^2S \tag{3}$$

where

$$C_L = C_L(M;\alpha,\delta_{VC1},\delta_{VC2})$$

$$C_D = C_D(M;\alpha,\delta_{VC1},\delta_{VC2}) \tag{4}$$

The drag polars presented in Figs. 1 and 2 may serve as examples for the realistic and complex characteristics applied in the numerical investigation described in this paper.

The thrust model accounts for the effect of speed, altitude, and thrust setting. It may be expressed as

$$T(V,h;\delta_T) = \delta_T T_{max}(V,h) \tag{5}$$

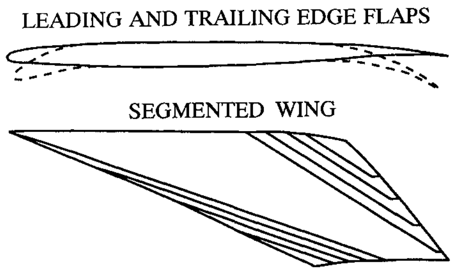


Fig. 3 Different techniques for varying camber (From Ref. 23).

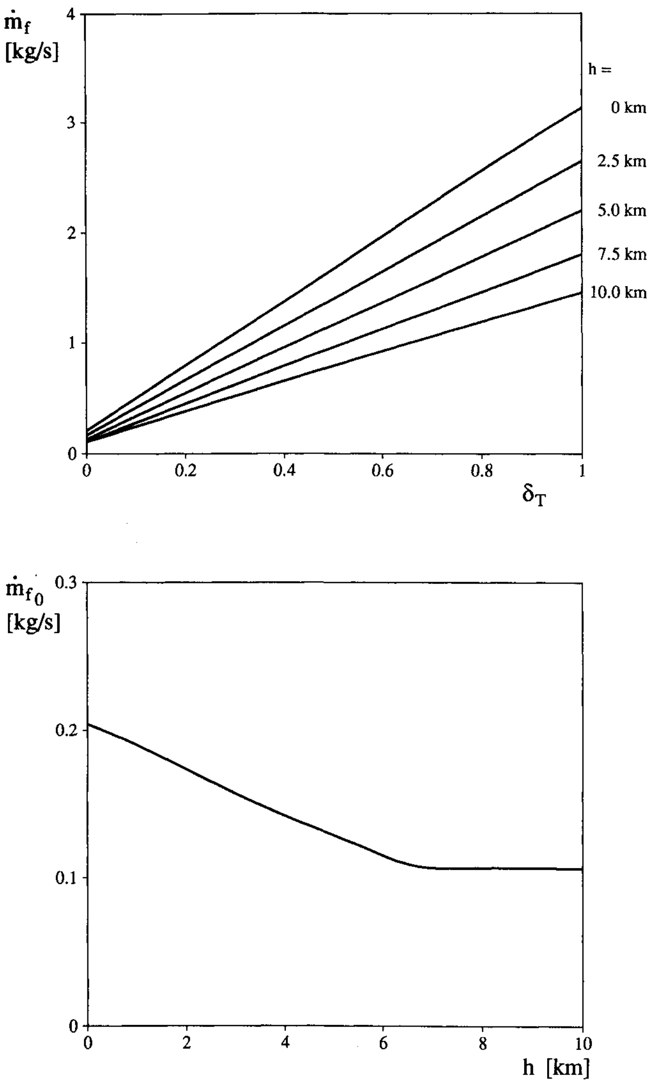


Fig. 4 Fuel consumption model.

Fuel consumption characteristics are described by the following relation:

$$\dot{m}_f = \dot{m}_{f_0}(h) + \delta_T \sigma(V) T_{\max}(V,h) \tag{6}$$

The mass of the aircraft can be considered constant for one period since the fuel consumed during such a time interval is small when compared with the total of the mass, i.e.,

$$m_f(1) - m_f(0) \ll m \tag{7}$$

Periodicity of the flight path implies the following boundary conditions:

$$V(1) = V(0), \qquad \gamma(1) = \gamma(0), \qquad h(1) = h(0) \tag{8a}$$

The initial condition for the fuel mass can be written as

$$m_f(0) = 0 \tag{8b}$$

Control variables are angle of attack α, variable camber setting δ_{VC1}, δ_{VC2}, and throttle setting δ_T that are subject to the following inequality constraints:

$$\alpha_{\min} \le \alpha \le \alpha_{\max}$$

$$(\delta_{VC1})_{\min} \le \delta_{VC1} \le (\delta_{VC1})_{\max}$$

$$(\delta_{VC2})_{\min} \le \delta_{VC2} \le (\delta_{VC2})_{\max}$$

$$0 \le \delta_T \le 1 \tag{9}$$

The atmospheric model used for describing air density, speed of sound, and thrust dependencies on altitude agrees with the ICAO Standard Atmosphere.[28]

The periodic control problem is to find the control histories α, δ_{VC1}, δ_{VC2}, and δ_T; the initial states $[V(0), \gamma(0), h(0)]$; and the optimal cycle length $\bar{t}_{cyc}$ that maximize the performance criterion $J = t_{cyc}/(m_f)_{cyc}$ subject to the dynamic system described by Eq. (2), boundary conditions given by Eqs. (8a) and (8b), and inequality constraints for the control variables, Eq. (9).

Optimality Conditions

Necessary conditions for optimality can be determined by applying the minimum principle. For this purpose, the Hamiltonian is defined as

$$H = t_{cyc}\left[\lambda_V\left(\frac{T}{m} - \frac{D}{m} - g\sin\gamma\right) + \lambda_\gamma\left(\frac{L}{mV} - \frac{g}{V}\cos\gamma\right)\right.$$
$$\left. + \lambda_h V \sin\gamma + \lambda_f(\dot{m}_{f_0} + \sigma T)\right] \tag{10}$$

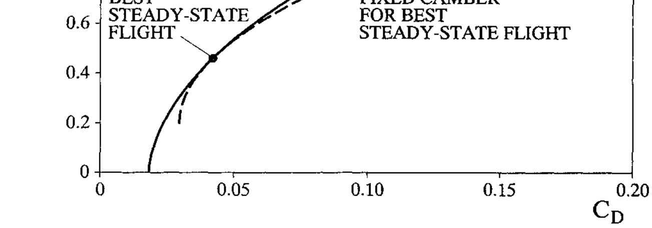


Fig. 5 Fixed camber drag polar for best steady-state endurance cruise and related periodic maximum endurance cruise.

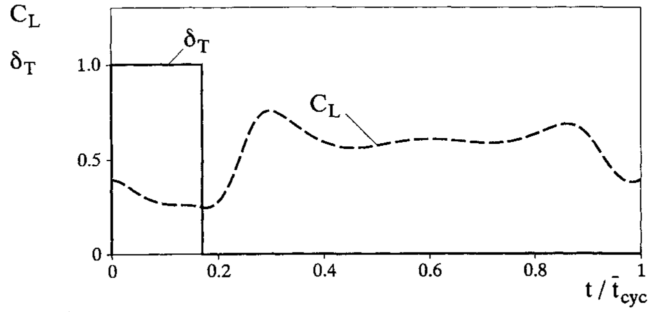


Fig. 6 Periodic maximum endurance cruise with fixed camber, control variables, 14.4% endurance increase, $\bar{t}_{cyc} = 246.4$ s.

where the Lagrange multipliers $\lambda = (\lambda_V, \lambda_\gamma, \lambda_h, \lambda_f)^T$ have been adjoined to the system of Eq. (2). The Lagrange multipliers (Partial derivatives of D, L, T are denoted by subscripts, e.g., $D_V = \partial D/\partial V$.) are determined by

$$\frac{d\lambda_V}{d\tau} = t_{cyc}\left[\lambda_V \frac{D_V - T_V}{m} + \lambda_\gamma \frac{L - VL_V - mg\cos\gamma}{mV^2}\right.$$
$$\left. - \lambda_h \sin\gamma - \lambda_f(\sigma_V T + \sigma T_V)\right]$$

$$\frac{d\lambda_\gamma}{d\tau} = t_{cyc}\left(\lambda_V g\cos\gamma - \lambda_\gamma \frac{g}{V}\sin\gamma - \lambda_h V\cos\gamma\right)$$

$$\frac{d\lambda_h}{d\tau} = t_{cyc}\left[\lambda_V \frac{D_h - T_h}{m} - \lambda_\gamma \frac{L_h}{mV} - \lambda_f\left(\sigma T_h + \frac{d\dot{m}_{f_0}}{dh}\right)\right]$$

$$\frac{d\lambda_f}{d\tau} = 0 \tag{11}$$

with the following boundary conditions:

$$\lambda_V(1) = \lambda_V(0), \qquad \lambda_\gamma(1) = \lambda_\gamma(0)$$

$$\lambda_h(1) = \lambda_h(0), \qquad \lambda_f(1) = -\frac{t_{cyc}}{m_f^2(1)} \tag{12}$$

The optimal cycle time $\bar{t}_{cyc}$ can be determined by applying two further differential equations

$$\frac{dt_{cyc}}{d\tau} = 0$$

$$\frac{d\lambda_t}{d\tau} = -\frac{H}{t_{cyc}} \tag{13}$$

subject to the boundary conditions

$$\lambda_t(0) = 0, \qquad \lambda_t(1) = \frac{1}{m_f(1)} \tag{14}$$

The optimal controls α, δ_{VC1}, δ_{VC2}, and δ_T are such that H is minimized. For this reason, α is determined either by (from $\partial H/\partial\alpha = 0$)

$$C_{L_\alpha}\frac{\lambda_\gamma}{V\lambda_V} - \frac{\partial C_D}{\partial\alpha} = 0 \tag{15}$$

or the constraining bounds of Eq. (9).

The condition for the optimal setting of variable camber yields

$$\lambda_\gamma \frac{\partial C_L}{\partial \delta_{VC1}} - \lambda_V V \frac{\partial C_D}{\partial \delta_{VC1}} = 0$$

$$\lambda_\gamma \frac{\partial C_L}{\partial \delta_{VC2}} - \lambda_V V \frac{\partial C_D}{\partial \delta_{VC2}} = 0 \tag{16}$$

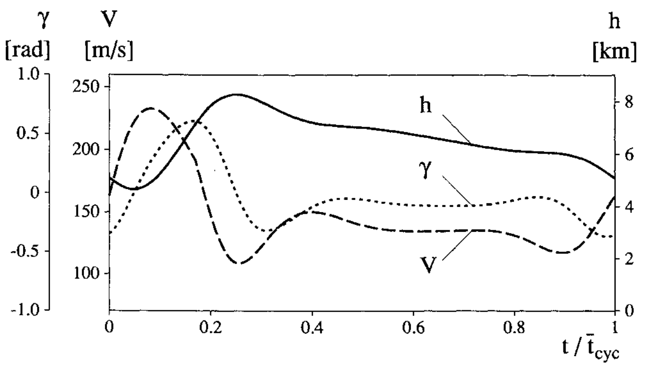


Fig. 7 Periodic maximum endurance cruise with fixed camber, state variables.

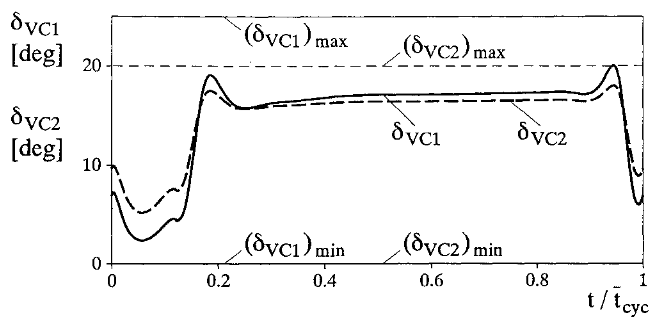


Fig. 8 Periodic maximum endurance cruise with optimal variable camber control, control variables (optimal variable camber setting), 25.4% endurance increase, $\bar{t}_{cyc} = 597.8$ s.

This relation applies when δ_{VC1} and δ_{VC2} are within their admissible sets. Otherwise, the constraining bounds of Eq. (9) become active.

With regard to throttle setting δ_T, the Hamiltonian shows a linear dependence. Accordingly, a bang-bang-type control or singular arcs can exist. The bang-bang-type control means that the throttle takes on its boundary values according to

$$\delta_T = 0 \quad \text{if} \quad \bar{S}(y,\lambda;\alpha,\delta_{VC1},\delta_{VC2}) > 0$$

$$\delta_T = 1 \quad \text{if} \quad \bar{S}(y,\lambda;\alpha,\delta_{VC1},\delta_{VC2}) < 0 \qquad (17)$$

where

$$\bar{S}(y,\lambda;\alpha,\delta_{VC1},\delta_{VC2}) = \frac{\partial}{\partial \delta_T} H(y,\lambda;\alpha,\delta_{VC1},\delta_{VC2},\delta_T) \qquad (18)$$

is the switching function and $y = (V,\gamma,h,m_f)^T$. Switching occurs when

$$\bar{S}(y,\lambda;\alpha,\delta_{VC1},\delta_{VC2}) = 0$$

A singular arc means that the throttle is not on its boundary, but takes on values interior to its admissible control set. This is the case when $\bar{S}(y,\lambda;\alpha,\delta_{VC1},\delta_{VC2}) = 0$ for a finite interval of t. However, such behavior was not observed in the numerical investigation.

It may be of interest to note that singular arcs have been found in recent investigations on periodic optimal performance problems.[29,30] They concern models with a performance index including time, range, and fuel or range cruise with specific engine characteristics.

The numerical difficulties existing in periodic optimal cruise problems require sophisticated optimization procedures and efficient computational algorithms. Such problems include the precise treatment of switching conditions, internal point and jump conditions, etc. The computer code applied is based on the method of multiple shooting and provides results with high accuracy.[31,32]

Results for Endurance Increase

The aircraft considered in the numerical investigation is realistically modeled, i.e., the functional dependencies account for all factors physically relevant for drag, thrust, and fuel consumption characteristics according to Eqs. (3-6) and (9) concerning the ranges admissible. Characteristics of the aerodynamics model are illustrated in Figs. 1 and 2, which show drag polars of the aircraft considered. For the powerplant system, fuel consumption characteristics are of particular importance for periodic optimal control problems. The model applied is illustrated in Fig. 4. The type of vehicle that has a wing loading $m/S = 345.5$ kg/m^2 can be regarded as representative of a high-performance aircraft capable of supersonic speed.

As a reference, a fixed camber setting is considered first. The setting chosen as a reference out of all fixed camber settings possible is such that it yields the best steady-state endurance cruise, i.e., the maximum of flight time per fuel consumed when performing a classical endurance cruise. The operating condition in regard to the drag polar is shown in Fig. 5.

For the reference case with a fixed camber setting, periodic optimal control already provides an endurance increase of 14.4% when compared with the best steady-state flight. The history of control and state variables is shown in Figs. 6 and 7 (with lift coefficient shown instead of angle of attack in order to provide a direct reference to the drag polar representations shown here). The high-thrust climbing-flight phase followed by an idle-thrust gliding-flight phase can be considered a basic characteristic of periodic optimal cruise. The lift coefficient is correlated with thrust such that it is reduced when the thrust setting is high and it is increased when the thrust setting is low. Speed varies in a manner corresponding to the changes of lift coefficient, i.e., it takes on comparatively large values when lift coefficient is reduced and vice versa.

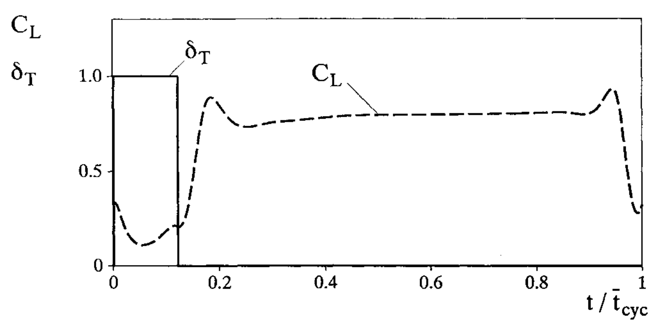


Fig. 9 Periodic maximum endurance cruise with optimal variable camber control, control variables (lift coefficient and thrust setting).

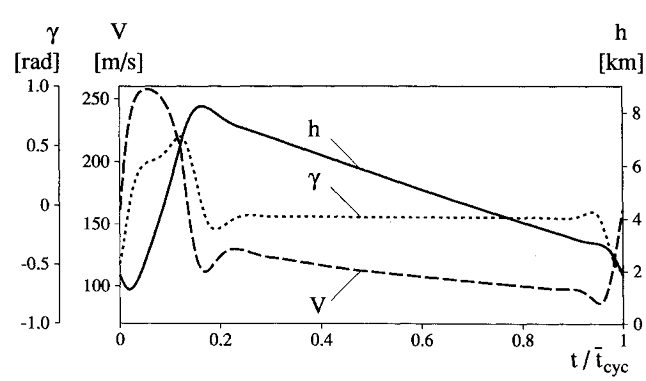


Fig. 10 Periodic maximum endurance cruise with optimal variable camber control, state variables.

Using camber as a control significantly enhances the improvement possible by periodic optimal control. Varying camber provides an endurance increase of 25.4% when compared with the best steady-state cruise. This is illustrated in Figs. 8–10, which depict the history of control and state variables.

The history of optimal camber control (Fig. 8) shows that this device is used to a significant extent. This becomes evident when comparing the variable camber settings applied during an optimal period with their admissible bounds, which are also indicated in Fig. 8. For the sinking flight phase at an idle thrust setting, wing camber is increased to provide the best lift/drag ratio at large lift coefficients. The opposite camber setting is applied in the climbing flight phase at small lift coefficients and high speed, respectively.

The two other control variables are presented in Fig. 9, with the lift coefficient shown (instead of angle of attack) for comparison purposes with the best fixed camber case considered previously. In the sinking flight phase, the lift coefficient values are now significantly larger than in the fixed camber case. A similar behavior holds for the climbing flight phase in regard to the smaller values attained. This is because varying camber improves the lift/drag ratio both in the higher and lower lift coefficient region.

In accordance with the controls, the state variables (Fig. 10) also show more pronounced changes during an optimal period when compared with the fixed camber case (Fig. 7). This holds particularly for speed and altitude. Flight-path angle shows smaller (negative) values during the sinking flight phase.

An insight into the physical mechanism underlying the aerodynamic performance enhancement due to optimal variable camber control can be provided by a presentation as in Fig. 11 that shows the lift/drag combinations attained during an optimal period. The curves depicted in Fig. 11 include the effect of Mach number on drag so that they do not represent actual drag polars, but lift/drag combinations at Mach numbers as they occur during an optimal trajectory. As may be seen, optimal variable camber control provides an improvement in the lift/drag ratio for both large and small lift coefficients. Accordingly, the lift coefficient range utilized during an optimal period can be extended when compared with the best fixed camber case.

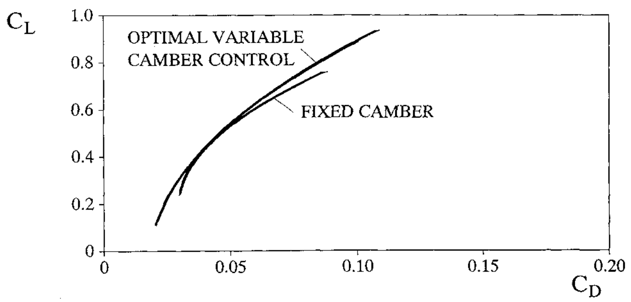


Fig. 11 Lift/drag combinations attained during an optimal period.

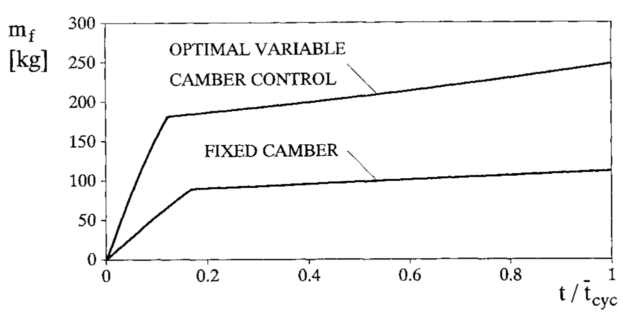


Fig. 12 Fuel consumption during maximum thrust and idling phases.

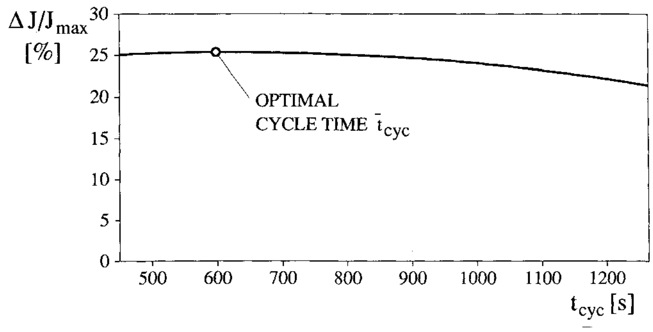


Fig. 13 Endurance increase per fuel consumed as a function of period length (J_{max}: best steady-state performance).

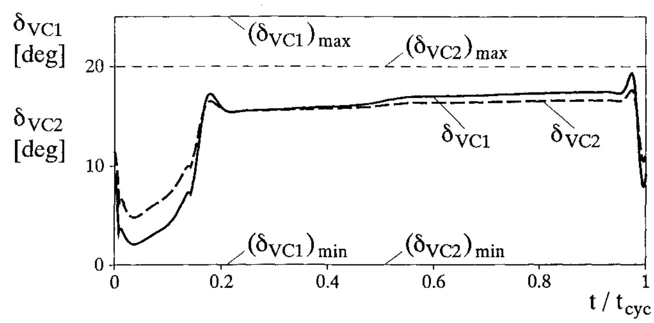


Fig. 14 Periodic endurance cruise with optimal variable camber control at double period length, control variables (variable camber setting), 22.2% endurance increase, $t_{cyc} = 1195.6$ s.

Engine Considerations

There are some topics that may be of interest in regard to engine operation. The engine is operated at maximum thrust and then changed to idle. Figure 12 shows that the fuel consumed in the idling phase cannot be ignored. This holds for optimal variable camber control, as well as the best fixed camber case. Despite this phase during which fuel is consumed without producing thrust, periodic optimal control provides a performance improvement when compared with the best steady-state endurance cruise.

During optimal periodic endurance cruise, the engine is alternately operated at high and low thrust settings. Such engine cycles may be disadvantageous in terms of engine lifetime considerations. Though an optimal period is of the order of 10 min (Figs. 8–10), an increase in the length of a period may be advantageous for reducing the number of engine cycles within a given total endurance time. Figure 13 shows how the performance criterion depends on the length of a period. As may be seen, only little change occurs even if the period length is doubled when compared with the optimal value.

The corresponding periodic flight profile is shown in Figs. 14–16. Basically, the altitude range is extended to enable longer periods. Thus, it is possible to increase the time for the maximum thrust phase, as well as the sinking phase at idling. Control of variable camber is similar to the optimal period case (Fig. 14). This also holds for the other controls and state variables as shown in Figs. 15 and 16.

As a consequence of altitude range extension, a constraint $h \geq h_{min}$ becomes active where $h_{min} = 0$. Additional conditions are necessary for treating this problem.[33] There may be two types of a solution in terms of how the trajectory is influenced by the constraint. One solution shows a point of contact where the trajectory touches the altitude limit h_{min} at one point only. The other solution concerns an arc on which the trajectory stays for a finite interval. In the numerical investigation, only the first type of solution was observed, so only it will be considered in the following treatment.

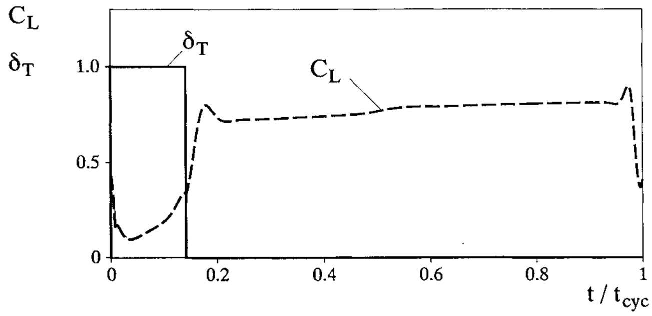


Fig. 15 Periodic endurance cruise with optimal variable camber control at double period length (lift coefficient and thrust setting).

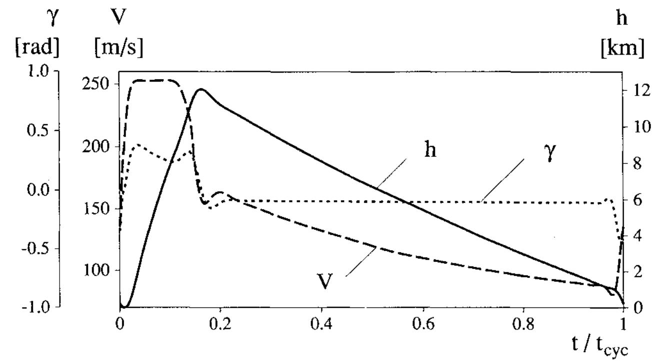


Fig. 16 Periodic endurance cruise with optimal variable camber control at double period length (state variables).

For treating the problem addressed, additional conditions may be formulated (according to Ref. 34). For this purpose, the relation

$$G(h) = h_{\min} - h \leq 0 \tag{19}$$

is introduced. Calculating successively the time derivatives of G until an expression is obtained that is explicitly dependent on the control variables results in

$$\frac{dG}{d\tau} = -t_{\text{cyc}} V \sin\gamma$$

$$\frac{d^2G}{d\tau^2} = -t_{\text{cyc}}^2 \left[\left(\frac{T}{m} - \frac{D}{m} \right) \sin\gamma + \frac{L}{m} \cos\gamma - g \right] \tag{20}$$

Accordingly, the state variable constraint is of the second order.

For the contact point problem, Eqs. (19) and (20) yield

$$h(\tau_1) = h_{\min} \quad \text{and} \quad \gamma(\tau_1) = 0 \tag{21}$$

where τ_1 denotes the normalized time at which the flight path touches the altitude constraint. At this point, λ_h shows a discontinuous change. If τ_1^- denotes the time just before the contact point and τ_1^+ the time immediately after it, the following relation holds:

$$\lambda_h(\tau_1^+) = \lambda_h(\tau_1^-) - \nu_0 \tag{22}$$

where $\nu_0 > 0$ is a constant.

Conclusions

A new method for maximizing the endurance of aircraft is proposed. This method consists of periodic optimal control of variable camber, in combination with the corresponding control of throttle and elevator.

It is shown that the proposed method significantly increases the maximum endurance of aircraft when compared with the best steady-state flight. This also holds when compared with periodic optimal endurance cruise at fixed wing camber, with this type of cruise yielding already an endurance increase in comparison to the best steady-state cruise. The improvements are due to the increased lift/drag ratios that variable camber provides both at large and small lift coefficients. At large lift coefficients, the sinking performance is improved. The advantageous effect of variable camber can be fully utilized for the gliding flight phase because it takes place at low Mach numbers. This is the Mach number region where variable camber is most effective in improving drag characteristics at high lift coefficients.

Engine considerations address the fuel consumed, with particular attention given to the idling phase. They are also concerned with an increase in the length of a period in order to reduce the number of engine cycles in regard to alternate operating at high and low thrust settings. Endurance performance shows a comparatively small reduction, even if the period length is doubled when compared with the optimal value.

The optimal control technique applied makes use of the minimum principle for deriving optimality conditions, with conditions for control and state constraints included. For the numerical investigation, a procedure based on the method of multiple shooting is applied. The aircraft considered is realistically modeled in terms of aerodynamics and powerplant characteristics. It may be considered representative of a high-performance vehicle capable of supersonic speed.

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